Optimal Static Potentials for Robust Macroscopic Quantum State Generation of Levitated Nanoparticles

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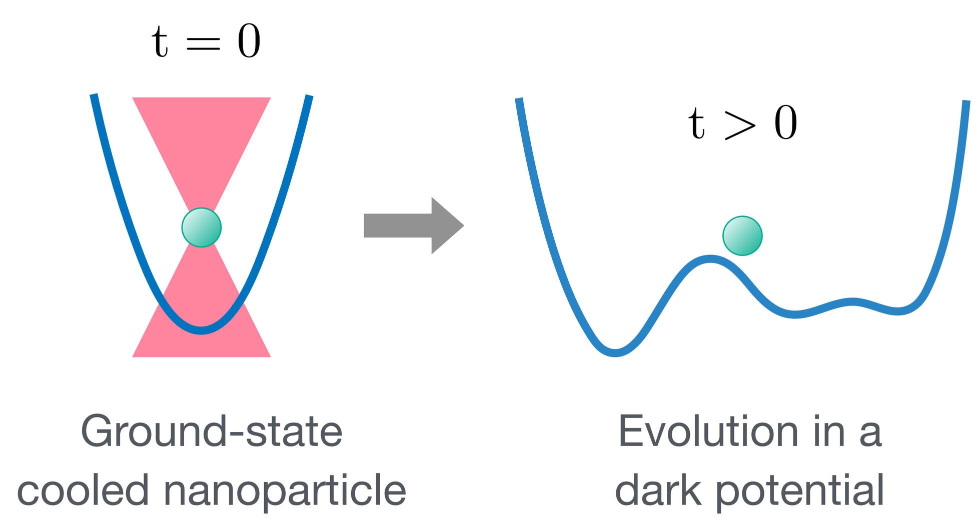
1. Introduction

❖ Levitated nanoparticles in the quantum regime

- Ground state cooling achieved [1,2,3]
- Towards preparation of **macroscopic quantum superposition states:**
  - Coupling to nonlinear systems is challenging
  - Evolution in dark nonharmonic potentials [4,5]
  - Need for delocalization to enhance quantum features, which introduces decoherence



What is the **optimal** potential shape?

❖ Proposal

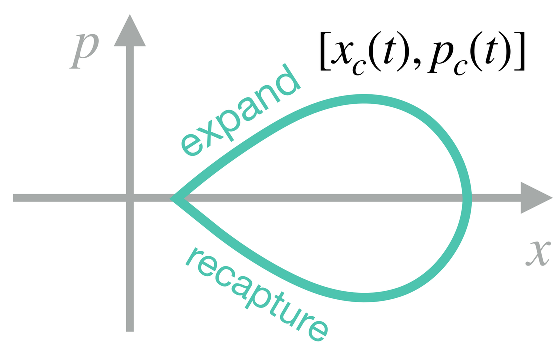
- Optimization of wide static potentials for the generation of macroscopic quantum states
- Optimization accounting for position-dependent **noise** sources within experimental setups
- Introduction of key **figures of merit** that allow for fast computation and capture macroscopicity and quantum signatures

2. Optimization

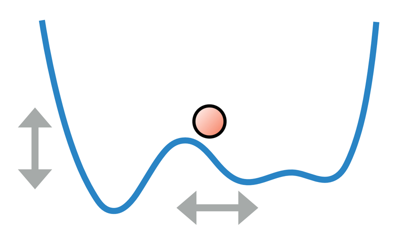
❖ Optimization of a static potential

$$V(x) = \frac{1}{2} m\omega^2 d^2 \sum_{n=1}^{N} \frac{c_n}{n!} \left(\frac{x}{d}\right)^n$$

Constraints:
- Closed classical trajectory
- Feasible potential
- Fast run



❖ Optimization in the presence of position-dependent noise



- Fluctuations in position and amplitude of the potential

$$\tilde{V}(x,t) = V\left(x + x_0 \xi_1(t)\right)\left[1 + \xi_2(t)\right]$$

$$\langle \xi_i(t) \xi_j(t') \rangle = 2\pi S_i \delta_{ij} \delta(t - t')$$

❖ **Figures of merit** based on Gaussian dynamics

- Classical trajectory frame: phase-space shift onto a trajectory associated with the classical dynamics [6]

$$\hat{H} = \frac{\hat{p}^2}{2m} + V_s(\hat{x}) \longrightarrow \hat{H}_c(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \sum_{n=2}^{N} \alpha_n(t) \hat{x}^n$$

$$\frac{\partial \hat{\rho}_c}{\partial t} = \frac{1}{i\hbar}\left[\hat{H}_C(t), \hat{\rho}_c(t)\right] - \frac{\Gamma_f(t)}{2X_\Omega^2}[\hat{x},[\hat{x},\hat{\rho}_c(t)]]$$

- Gaussian dynamics

$$\hat{H}_G(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \alpha_2(t) \hat{x}^2 \longrightarrow$$ squeezing with parameter $\eta(t)$ and angle $\phi(t)$

- First quantum correction

$$\hat{V}_{NG}(t) \simeq \frac{1}{2} m\omega^2 \alpha_3(t) \hat{x}^3 \longrightarrow \hat{V}^G_{NG}(t) \simeq \beta_3(t)\left[\hat{x}\cos(\phi(t)) + \hat{p}\sin(\phi(t))\right]^3$$

- Figures of merit that allow for fast computation

- Coherence length: $\xi = \sqrt{8\langle \hat{x}_\phi^2 \rangle}\mathscr{P}(t)$ $\quad \langle -\frac{\hat{x}_\phi}{2} | \hat{\rho}_G | \frac{\hat{x}_\phi}{2} \rangle \propto \exp\left(-x^2/\xi_\phi^2\right)$

- Cubicity and purity:

$$K(t) = \kappa_3(t)\mathscr{P}(t) \qquad \kappa_3^2(t) = \left[\int_0^t dt' \beta_3(t') \sin(\phi(t'))\right]^2 + \left[\int_0^t dt' \beta_3(t') \cos(\phi(t'))\right]^2$$

3. Example: quartic potentials

❖ Static quartic potential

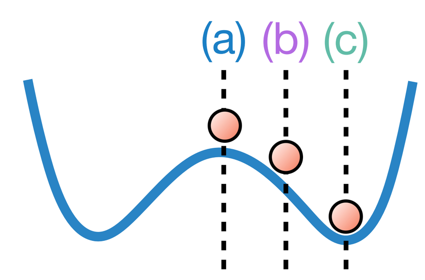
$$V(x) = \frac{1}{2} m\omega^2 \left(a(x - d_0)^2 + \frac{b}{2d^2}(x - d_0)^4\right)$$

a, b, d_0 optimization parameters

❖ Optimization of coherence length

- Optimal configurations:

$$\Gamma_f(t) = \frac{2\pi X_\Omega^2}{\hbar^2}\left(S_1 X_\Omega^2 [V''(x_c(t))]^2 + S_2 [V'(x_c(t))]^2\right)$$



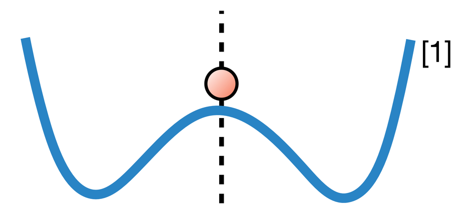
(a) Small noise $(S_i\omega \lesssim 1, i = 1,2)$

(b) Large displacement noise $(S_1\omega \gtrsim 10)$

(c) Large intensity noise $(S_2\omega \gtrsim 10)$

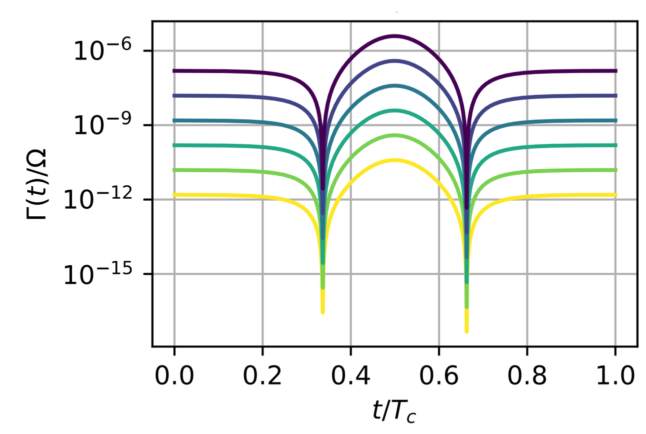
❖ Optimization of cubicity and purity

- Optimal potential:



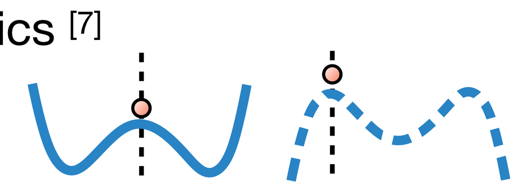
$a = -1, b = 1, d_0/d \to 0$

- Decoherence rate:

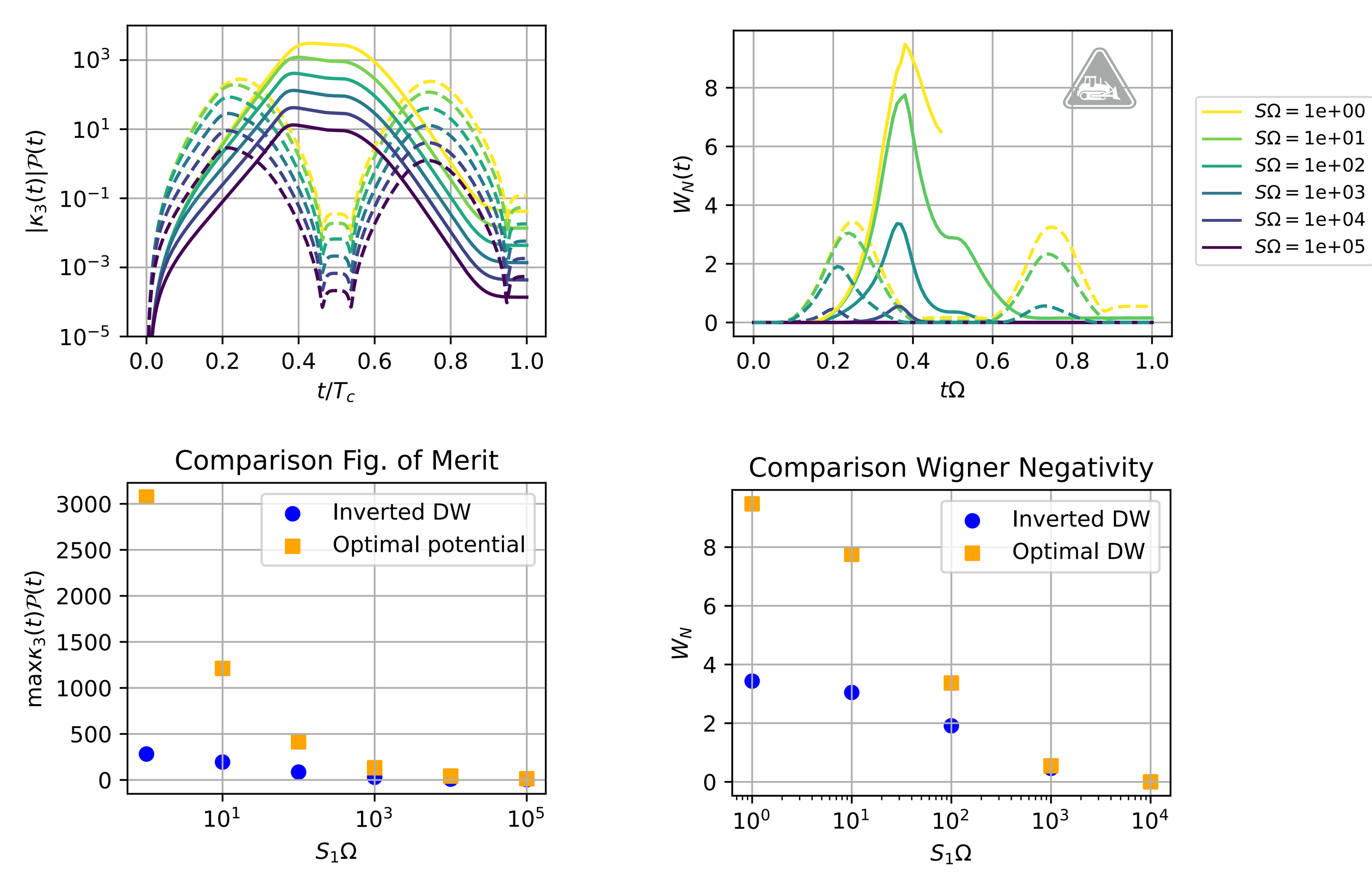


❖ Comparison with full quantum dynamics [7]

- Two potentials: DW and inverted DW



- Our figure of merit qualitatively captures the Wigner negativity features



4. Conclusion and Outlook

❖ We propose a new method for optimizing static potentials for the generation of macroscopic quantum states of levitated nanoparticles in the presence of noise.

❖ We introduce figures of merit that allow for fast computation and capture macroscopicity and quantum signatures.

❖ We obtain the optimal quartic potential for maximizing squeezing and for generating quantum cubic states.

❖ **Outlook:**

- Optimization within more general families of potentials
- Optimal potentials for noise metrology

5. References

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